AN INVESTIGATION OF STATISTICAL REASONING SKILLS OF MIDDLE SCHOOL STUDENTS ABOUT DISTRIBUTION

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This study aims to examine the informal statistical reasoning processes students perform to sense and comprehend the distribution. In two activities that we applied to eight students in the 11-13 age range, we examined students' statistical reasoning skills throughout their modeling processes to help them develop a foundation. We observed that students were able to create distribution displays in the activities that we applied, to express some statistical concepts, and to elaborate and analyze distribution representations. We explained the modeling process that students experienced during the activities, according to the RISM framework. One of the significant results that we have obtained is that when students can see the distribution as a whole, they realize the spread of the distribution, the center of the distribution, the variability of the distribution, and their probabilistic thinking increases when they express it through various samples.

INTRODUCTION

Although being the science of learning from data, statistics include data, data analysis, and statistical inference (Moore, 2005), and statistics teaching is increasingly based on content using authentic activities that are important to students (Wild et al., 2011). The skeleton structure in which the statistical context is chosen in terms of preparing the authentic environment and determining factual aspects in statistical research will provide a common language for researchers to better express statistical reasoning (Makar et al., 2011).

The approaches and goals specified in recent studies have produced various frameworks to identify them by addressing students' pedagogical behavior between statistical modeling and statistical reasoning (Pfannkuch et al., 2018). Offering a common approach to statistical modeling activities, the Exploratory Data Analysis (EDA) lacks probabilistic assessments that are important for understanding the relationship between sample and population. The probability approach, another approach that emphasizes modeling activities on how statisticians use probability in problem-solving, lacks the steps of real data research, such as asking a research question. The integrated modeling approach (IMA) combines these two approaches with modeling-based guidelines. IMA was developed to guide the design and analysis of experimental tasks, to deepen their modeling when performing informal statistical inference (ISI), and to guide the evaluation of this reasoning (Braham & Ben-Zvi, 2017).

In recent years, statisticians have made various comments to distinguish between mathematics and statistics, keep statistics in a separate discipline area, and draw attention to their differences. Unlike mathematical reasoning, statistical inquiry depends on data and is typically based on context (Cobb & Moore, 1997; Moore, 1998; Pfannkuch & Wild, 2000; Wild & Pfannkuch, 1999). Informal inferential reasoning (IIR) is defined as making generalizations about a population from random samples and formulating it in the cognitive process using informal statistical tools (Makar et al., 2011).

Jones et al. (2004) express the term cognitive development model in statistical reasoning as a theory suggesting different growth levels or growth models resulting from maturation or interactive effects in structured and unstructured learning environments. A model can be thought of as a conceptual system (mental model) that supports creating meaning in a particular setting (Lesh & Doerr, 2003). If a model was created for a statistical purpose, it should have two distinctive features: The first one is that phenomenon that is tried to explain includes variability, and the second is the existence of probabilistic predictions and thoughts to use variability (Brown & Kass, 2009). If modelers to be considered are young students, probabilistic thinking here brings informal statistical reasoning. Since probability feeds uncertainty and is the state of measuring uncertainty, informal reasoning is not precise is indispensable (Dvir & Ben-Zvi, 2018).

In: P Arnold (Ed.), *New Skills in the Changing World of Statistics Education*
Proceedings of the Roundtable conference of the International Association for Statistical Education (IASE),
July 2020, Online. Voorborg, The Netherlands. iase-web.org/Conference_Proceedings.php

The approaches and goals specified in recent studies have produced various frameworks to identify them by addressing students' pedagogical behavior between statistical modeling and statistical reasoning (Pfannkuch, Ben-Zvi, & Budgett, 2018). One of the theoretical frameworks in which students are essential as modelers are Reasoning with Informal Statistical Models and Modeling (RISM). This framework, created by Dvir and Ben-Zvi, advocates statistical modeling as alternative modeling while covering statistical reasoning. In this framework, it is particularly useful to divide the informal modeling process into three separate modeling processes, though not independent: modeling a conjuncture, modeling data, and comparing it to a comparison model (Dvir & Ben-Zvi, 2018). The concept of the statistical modeling process proposed by Dvir and Ben-Zvi (2018) into the sub-modeling processes also forms the basis of the model comparison framework.

According to Bakker and Gravemeijer (2004), the variability of the data is both conceptual and practical. While a conceptual point of view, which concepts are supported by distributions and why these concepts are essential, the applied perspective focuses on how the data collection captures, displays, and processes. Students can explore relevant contexts by understanding how different distribution structures built on the same data are related. Therefore, distributions are conceptual, organizational, or mental tools that allow the development of a statistical intellectual method. These structures are complex and thin and require a culture to understand them. Thus, many questions arise about the conceptual, pedagogical, and research related aspects of reasoning about distributions (Pfannkuch & Reading, 2006). This research will examine how students perform the reasoning process by modeling what statistics (mean and median) are in the data distribution. Besides, the comments they made about the population on the sample will be compared based on the data distributions created by the students. Since the models are the key to statistical thinking, they will make students think more deeply about the concepts of distribution, variability, and finding the center at an earlier age. Through this study, the following research question was tried to be investigated: "How do we evaluate the distributions made by middle school students through the statistical modeling process in terms of their statistical reasoning?"

METHODOLOGY AND DATA COLLECTION PROCEDURE

This qualitative study was designed as a case study. Case studies focus on a specific topic, event, or situation, and the data collection process is enriched using multiple data collection tools such as observations, interviews, documents, audiovisuals (Creswell, 2007).

This study is a part of a more extensive study consists of activities that are applied to seven students between the ages of 11-13 (6^{th} and 7^{th}-grade levels) in a computer lab for six weeks and the learning process of TinkerPlots. In this study, two activities were analyzed. The computer screen recorder program, student worksheets, researchers' field, and observation notes were used for data collection during the activities.

Activity 1: Mystery Mixer

In this activity, where the students met the sampler tool, a dataset with numbers between 56 and 124 is given. We asked students to find the aggregate zone of the data by keeping the sample size to a minimum and using the divider tool. Besides, we observed how students used the divider to determine sample size to examine how they understood the different possibilities of the sample.

Activity 2: Fish-Length Distribution

Students were expected to decide whether to buy a genetically modified fish from a fish farmer in this activity. There are two different fish groups: the fish farmer's fish and genetically modified fish. Students will compare the length of the fish in the two groups. In the second part of the activity, we asked students if samples of 15 fish first and 130 fish were sufficient to conclude that genetically modified fish were longer than normal fish. At the latest stage, we expect them to confirm their answers, making similar inferences with the entire population. We aim to enable participants to see the relationship between population and sample over distribution and to explain it by associating it with the variability and center of a distribution.

FINDINGS

This study includes findings from two activities with students. In these activities, we examine how the students make inferences and informal inferential reasoning processes by comparing the conjecture model and data model. In the first activity, we determined the students' conjecture models to decide the sample size through the sampler tool they encountered for the first time. The student who determined the sample size for each activity according to the distribution it generated was Selim; the students who did not change the same sample size were Reyhan and Halit. Utku modeled the sample size at random. Students created data models for four different mixers. Because they use the sampler tool in this activity, the probabilities of distributions modeled by the students vary. However, although their samples were different, the distributions they would get for each mixer were normal, left-skewed, normal, and right-skewed, respectively. While only Selim discovered these distribution displays in data models, Yaman progressed through the modeling process by estimating the distributions' aggregate intervals with the divider tool. Other students continued the data modeling process to explore the aggregate regions of the data. The students inferred by comparing sample size and data modeling process. Fatih has divided into ten regions while determining the aggregate intervals with the divider tool during modeling. Reyhan also estimated the aggregate intervals by dividing the distribution of the data into four parts for each sample. As Selim was able to discover the types of distributions, he discovered the normal distribution displays with fewer regions, and the left- and right-skewed distributions with more regions.

In the second activity, students inferred the variability of the distribution by comparing them through sample-population relationships, centers of distributions, and interpreting the differences between them, in order to understand the variability of the distribution. For 130 fish, the students' answers to whether normal fish or genetically modified fish are longer forms the conjecture models. In these models, we could say that some students still do not understand the distribution because they respond with their ideas. Aylin, Hakan, Utku, and Reyhan prepared a distribution display to answer the given research question; Fatih, Yaman, Halit answered with a categorical display. The students who made up the distribution displays could see that the genetically modified fish were longer by answering the research question correctly. Only Fatih formed both distributions and claimed that normal fish were longer. None of the other students could answer this question that genetically modified fish are longer. In this part of the activity, it was vital for us how students noticed the variability in the distributions they generated. Except for Fatih, other students discovered the distribution concerning variability, but in doing so, explained it with the conjecture model. Reyhan modeled the distribution displays of the data sets based on sample sizes to make the center the same.

We examined the inferences the students made when they compared the distributions for 15 and 130 fish with all the fish in the lake. Aylin made this comparison with variability. She said that the center for 15 fish regularly changed, the center for 130 fish moved less, and the center for 625 fish did not change at all in each random distribution. Halit decided based on the aggregate intervals of the distributions he formed with different sample sizes. Halit said that while he selected 130 fish in response, 15 fish did not form any distribution. However, when he examined the population, he responded similarly to Aylin. Hakan, who was interested in the displacement of the center of the resulting distributions, realized that there were no distributions with 15 fish such as Halit, and discovered that, like Aylin, the center of each different distribution was also different. However, Hakan, unlike other students, was interested in the distance of the median and mean to each other in the distributions he obtained for 15 fish. Reyhan was the one who could not produce any idea of the relationship between the sample and population, the meaning of the distribution display she generated, or the difference in variability among the students.

DISCUSSION

The data collection process revealed the inferences students encountered in their modeling processes with real-life data. In this section, we discuss the contributions or differences we can make to the RISM framework based on the purpose of the activities and how the students' resulting reasoning processes are finalized by modeling.

The mystery mixer activity is an activity where we think students are discovering distributions. In this activity, we found that students construct conjecture models, sample size, representation of distribution, the spread of data, and randomness. Bakker (2004) emphasized *grown samples* in his study that as participants increased the sample size, they made better estimates for the population relative to decreasing variability. However, when the student creates the distribution with the sampler tool, the feeling about the population also occurs when estimating the region where the changing center is located since we observed how they used the divider tool in the informal reasoning process.

In the fish-length distribution activity, we observed how students comprehend the distributions during the reasoning process they went through. We noticed that data models do not help students inference for variability; they explain it with conjecture models. We understood this from the students' statements that there are unbalanced data on variability, such as increase and decrease. In their study Lehrer and Schauble (2004) investigating natural variation, the result that children seek *plateaus* as low variability indices matched the findings we have obtained and supported our claim. When we examine the process of creating data models, we could claim that as students discover the distribution, the number of TinkerPlots tools they use increases, and they try to understand the distributions they create. We noticed that students inferred variability of distributions, the center of distributions, or mean and median values, by the sample-population relation of distributions.

All of these results enabled us to explore with the RISM framework how students perform informal reasoning processes for statistical concepts in empirical distribution building processes. The discovery is that students can make sense by exploring fundamental concepts such as variability, center, and spread, supported by various activities concerning informal reasoning for distribution, so that they can create theoretical distributions in the future.

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